Ministry of Finance
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Forecasting VAR Analysis for a DSGE-VAR Model

Kateřina Gawthorpe

Metohodological Compendium

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The Methodological Compendium presents the methodological approaches of the Ministry of Finance of the Czech Republic in the areas of macroeconomic and fiscal analyses, forecasts and projections.

We will gladly welcome relevant comments or suggestions to improve the publication. Please send any comments to the author of the publication.

Introduction

The Czech Ministry of Finance applies a complex DSGE-VAR model for forecasting purposes. This study concentrates on a presentation of the VAR component which is estimated separately for every macroeconomic variable subject to prediction.

DSGE models are well-known for their compelling long-run predictions while VAR models are prevalent tools for economists focusing on short-run and medium-run forecasts. Final prediction incorporates different versions of VAR models along with an outcome from DSGE-BAYES forecasts with respect to their forecasting accuracy. The final model combines advantages of Bayesian estimation with benefits of regression analysis.

Forecasting performance for VAR models is evaluated based on an iterated rolling-window forecast always estimated one period ahead for forty periods in quarterly frequency from the first quarter of 2012 to the second quarter of 2019. Forecast for the DSGE part of the model is provided by Dynare, the Bayesian method of estimation is designed to secure perfect fit of observable variables on data, this translates into alignment of the modelled series and the historical series. The DSGE model is an extended version of Aliyev et al. (2014).

Final out-of-sample forecast is an outcome of the DSGE-VAR estimation. Selection of suitable VAR models for prediction depends on economic theory, statistical significance of individual variables in the model as well as on a size of root mean squared errors (RMSE). While increasing number of variables is common way to boost in-sample forecast fit, it negatively affects the out-of-sample forecast. Several studies (Taiwo & Olatayo, 2013; Hafer & Sheehan, 1989; Lack, 2006; faculty.washington.edu) prefer simple VAR processes with minimum lags and only few variables to operate as sufficient forecasting methods. For similar reasons, we apply only one lag in our VAR models used for forecasting purposes.

1 Model Selection

Selection of a suitable model for forecasting an evolution of particular variable is based on economic theory, statistical significance of variables in the model and sizes of root mean squared errors.

Set of models used for estimation in this study starts with vector autoregressive models with exogenous variables, VARX. The vector autoregressive model captures mutual short-run relationships among multiple variables. The x suffix in the VARX name reflects exogenization of one or more variables in the model. This type of model estimates an effect of predetermined or exogenous variable on other variables in a VAR model. It enables to introduce exogenous forecasts into the prediction process of a variable. The VARX model is very appropriate for a small open economy such as the Czech Republic where we can incorporate exogenous forecasts of European economies estimated with different models or as published by foreign institutions.

Number of variables and subsequently unknown parameters commonly improves in-sample forecast but exacerbates a model's out-of-sample forecasting performance. The next utilized model attempts to overcome such shortcoming. Bayesian vector autoregressive model BVAR reduces unknown knowledge about parameters which assists to boost its forecasting quality. A researcher introduces a prior knowledge about a parameter; we apply so-called Minnesota priors. Minnesota priors are based on an assumption that most variables follow an AR process with an AR(1) parameter close to one. Next, these priors assume that lags of other variables are less informative than own lags and finally more recent lags are expected to be more informative than more distant lags.

Similarly to the BVAR model, factor augment vector autoregressive model FAVAR tends to outperform small models as it enables to capture more information about multiple relationships among variables. This class of models is designed for large datasets, where common latent variables are assumed to drive most variables. Computation of these common variables enables to reduce number of unknown parameters in a model which subsequently improves an out-of-sample forecasting accuracy of the model. The FAVAR model is then also a suitable tool for modelling economic cycle. The idea behind FAVAR assimilates DSGE modelling where a few shocks drive most variables. We specifically look for three so-called principal components which describe about fifty percent of data variation. Number of variables is thus reduced to three series which copy the common pattern of series visible on data.

Finally, the last model is vector error correction model VECM. The VECM estimates both short-term and long-term effects of one time series on another for the Czech economy. This capturing of long-run and short-run relationships among variables is an obvious advantage of this model. Significant long-run relationship demonstrates an existence of equilibrium in an economy. It is important to base selection of suitable variables on economic theory, long-run relationships among variables should correspond to their steady-states. Next, the error correction part also estimates speed of return of a dependent variable to equilibrium after a change in another variable.

2 Data

The used dataset originates in the Czech Ministry of Finance database and the Czech Statistical Office. The only exception concerns time series for the GDP of the European Union (EU), the money market interest rate for the EU and the money aggregate M2 for the Czech economy. The data for the GDP of the EU and the interest rate for the EU are collected from the dataset published by the Eurostat. The M2 variable is publicly available on the Czech National Bank website. All data are available with a quarterly frequency from 2000 to the first quarter of 2019. Data without seasonal adjustment are seasonally adjusted with the X-11 ARIMA method. Such time series subject to seasonal adjustment include: labour supply, labour demand, capital stock and consumer prices.

Subsequently, the data are transformed to year-on-year growth rates. Individual variables are in a logarithmic form with the exception of the Interest rate (3-month PRIBOR), the Interest rate for the EU, individual price deflators and the utilization rate calculated as the difference between capital demand and capital stock (fixed assets). The next table summarizes basic statistics for the chosen variables.

Table 1: Summary Statistics

		Mean	Median	Max.	Min.	Std. deviation
GDP (*Y*)	*y/y %*	2.8	2.9	7.2	-5.6	2.9
GDP of the EU (*Y EU*)	*y/y %*	2.8	3.2	6.2	-4.5	2.1
Export (*X*)	*y/y %*	8.1	7.4	37.1	-16.9	9.1
Consumption (*C*)	*y/y %*	2.2	2.7	5.8	-1.7	1.7
Import (*M*)	*y/y %*	7.6	7.1	34.8	-17.0	8.4
Government consumption (*G*)	*y/y %*	1.7	1.7	9.0	-5.1	2.9
Investment (*I*)	*y/y %*	2.9	4.0	20.3	-20.6	8.5
Exchange rate (*S*)	*y/y %*	-1.7	-2.0	8.0	-13.7	4.9
Interest rate (*IR*)	*%*	1.9	1.8	5.4	0.3	1.4
Price index (*PC*)	*y/y %*	1.1	0.9	4.0	-1.8	1.3
Wage (*W*)	*y/y %*	4.7	5.0	9.8	-1.5	2.8
Labour demand (*LD*)	*y/y %*	0.6	0.9	2.3	-3.2	1.3
Labour supply (*LS*)	*y/y %*	0.3	0.3	1.9	-1.3	0.7
Capital stock (*KS*)	*y/y %*	1.3	1.3	2.4	0.7	0.4
Capital (*K*)	*y/y %*	1.4	1.5	3.6	-4.1	1.4
M2 aggregate (*M2*)	*y/y %*	6.8	6.1	13.3	2.3	2.8
Price of export (*PX*)	*y/y %*	-0.2	0.2	7.6	-7.6	3.0
Price of import (*PM*)	*y/y %*	-0.4	0.3	4.9	-10.3	3.4
Price of investment (*PI*)	*y/y %*	0.8	1.0	3.8	-2.4	1.4

Source: Author's calculations.

3 VAR Models

This section provides an outcome only for the most suitable models as based on economic theory and measured by the size of Root Mean Square Errors.

3.1 Gross Domestic Product Models

3.1.1 VECM Model

The vector error correction model tests a potential short-term but also long-term relationship between the series GDP Y_t and labour demand L_t.

General form of this model is:

$$\begin{bmatrix}\Delta Y_t\\ \Delta L_t\end{bmatrix} = \begin{bmatrix}\psi_1\\ \psi_2\end{bmatrix} + \begin{bmatrix}\alpha_1\\ \alpha_2\end{bmatrix}[L_{t-1} - \beta_0 - \beta_1 Y_{t-1}] + \begin{bmatrix}\psi_{1,1} & \psi_{1,2}\\ \psi_{2,1} & \psi_{2,2}\end{bmatrix}\begin{bmatrix}\Delta Y_{t-1}\\ \Delta L_{t-1}\end{bmatrix} + \begin{bmatrix}\varepsilon_{1t}\\ \varepsilon_{2t}\end{bmatrix}. \quad (1)$$

The estimation of the above equation provides following outcome:

$$\Delta Y_t = -0.132\,(L_{t-1} - \mathbf{0.496}\boldsymbol{Y_{t-1}} - 5.103) - 0.369\Delta L_{t-1} + \mathbf{0.543}\Delta \boldsymbol{Y_{t-1}} + \mathbf{0.002} \quad (2)$$

[-1.150] [-6.172] [-1.277] [3.346] [1.719]

R^2=0.39,

$$\Delta L_t = \mathbf{-0.094}\,(L_{t-1} - \mathbf{0.496}\boldsymbol{Y_{t-1}} - 5.103) + 0.169\Delta L_{t-1} + \mathbf{0.218}\Delta \boldsymbol{Y_{t-1}} + 0.0004 \quad (3)$$

[-2.038] [-6.172] [1.461] [3.357] [0.667]

R^2=0.58.

This model was originally developed for forecasting labour demand, but its forecasting accuracy proves it to be suitable also for explaining variation of data for the GDP variable. The Johansen's cointegration test but also the significance of the error correction term proves the series of output and labour demand to share a common stochastic trend. The intercept in the error correction bracket signals a similar evolution of the two series which in the long-run differ only by a constant number. In the long-run mainstream economists commonly assume money-neutrality which cancels out any potential effects of nominal variables. The long-run evolution of the economic production is then defined by technological progress or other real effects. Productivity of labour is often used as a proxy for technological growth plus labour is often understood as a factor of production. From this perspective, it is intuitive to find long-run link among the labour variable and the GDP.

3.1.2 FABVAR Model

The next tested model is a factor-augment Bayesian vector autoregressive model FABVAR. The estimation of factors along with the Bayesian character of the model enable us to use large set of time series without reducing an out-of-sample forecasting accuracy. The model was initially developed for forecasting investment variable but proved to be beneficial also for forecasting GDP. $YF1_t$, $YF2_t$ and $YF3_t$ represent principal components which reflect a common trend for selected time series. Specifically, the factors combine data from variables such as consumption, government consumption, export, import, M2, labour demand, labour supply, wage, price of investment and capital demand. The Bayesian estimation assumes Minnesota priors for parameters; this approximation delivers a more accurate out-of-sample forecast than if we apply a simpler FAVAR model. These priors benefit from favourable characteristics as described in the beginning of this study.

Giannone et al (2012) also recommend these priors for providing accurate out-of-sample predictions on macroeconomic data. Sims (1992a cited in Giannone et al., 2012) finds VARS to exhibit a temporal heterogeneity where series tend to behave differently in the beginning and at the end of their sample. Minnesota priors reduce the importance of this deterministic component (expectation of future value for a variable given its initial conditions) which helps to improve a forecasting ability of a VAR model.

General representation of the model takes on the subsequent form:

$$\begin{bmatrix} \Delta Y_t \\ \Delta I_t \\ \Delta YF1_t \\ \Delta YF2_t \\ \Delta YF3_t \end{bmatrix} = \begin{bmatrix} \beta_{1,0} \\ \beta_{2,0} \\ \beta_{3,0} \\ \beta_{4,0} \\ \beta_{5,0} \end{bmatrix} + \begin{bmatrix} \beta_{1,1} & \beta_{1,2} & \beta_{1,3} & \beta_{1,4} & \beta_{1,5} \\ \beta_{2,1} & \beta_{2,2} & \beta_{2,3} & \beta_{2,4} & \beta_{2,5} \\ \beta_{3,1} & \beta_{3,2} & \beta_{3,3} & \beta_{3,4} & \beta_{3,5} \\ \beta_{4,1} & \beta_{4,2} & \beta_{4,3} & \beta_{4,4} & \beta_{4,5} \\ \beta_{5,1} & \beta_{5,2} & \beta_{5,3} & \beta_{5,4} & \beta_{5,5} \end{bmatrix} \begin{bmatrix} \Delta Y_{t-1} \\ \Delta I_{t-1} \\ \Delta YF1_{t-1} \\ \Delta YF2_{t-1} \\ \Delta YF3_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \varepsilon_{3t} \\ \varepsilon_{4t} \\ \varepsilon_{5t} \end{bmatrix}. \quad (4)$$

Substitution of the parameters with estimated values yields following results:

$$\Delta I_t = -0.773 + \mathbf{0.661 \Delta I_{t-1}} + 0.571 \Delta Y_{t-1} + 0.001 \Delta IF1_{t-1} \quad (5)$$

[-0.655] [3.948] [1.072] [0.132]

$$+0.0004 \Delta IF2_{t-1} + 0.0001 \Delta IF3_{t-1}$$

[0.362] [0.260]

R^2=0.67,

$$\Delta Y_t = 0.171 - 0.006 \Delta I_{t-1} + \mathbf{0.894 \Delta Y_{t-1}} - 0.001 \Delta IF1_{t-1} \quad (6)$$

[0.605] [-0.155] [7.083] [-0.739]

$$-7.338e^{-5} IF2_{t-1} + 0.0001 \Delta IF3_{t-1}$$

[-0.245] [0.860]

R^2=0.80.

This model yields higher R-squared than the previous one. However, the significance of individual variables is lower.

3.1.3 FABVARX Model

The FABVARX model as a factor augmented Bayesian vector autoregressive model with exogenous variable benefits from a high-quality forecasting performance while taking into account large set of variables. The Bayesian character of the model adds to the accuracy of the model estimation as it reduces the uncertainty concerning estimated parameters. Finally, the GDP of the European Union Y_t^{EU} enters the model as an external variable. The principal components in this model combine the same time series as in the previous one. Again, the principal components which explain more than fifty percent of the variation of the data enter the final FABVARX model. Alike the previous model, the Bayesian estimation of this model assumes the parameters of individual variables to be best represented as the so-called Minnesota priors. The model in the general form appears as follows.

$$\begin{bmatrix} \Delta Y_t \\ \Delta IR_t \\ \Delta S_t \\ \Delta \Pi_t \\ \Delta YF1_t \\ \Delta YF2_t \\ \Delta YF3_t \end{bmatrix} = \begin{bmatrix} \beta_{1,0} \\ \beta_{2,0} \\ \beta_{3,0} \\ \beta_{4,0} \\ \beta_{5,0} \\ \beta_{6,0} \\ \beta_{7,0} \end{bmatrix} + \begin{bmatrix} \beta_{1,1} & \beta_{1,2} & \beta_{1,3} & \beta_{1,4} & \beta_{1,5} & \beta_{1,6} & \beta_{1,7} \\ \beta_{2,1} & \beta_{2,2} & \beta_{2,3} & \beta_{2,4} & \beta_{2,5} & \beta_{2,6} & \beta_{2,7} \\ \beta_{3,1} & \beta_{3,2} & \beta_{3,3} & \beta_{3,4} & \beta_{3,5} & \beta_{3,6} & \beta_{3,7} \\ \beta_{4,1} & \beta_{4,2} & \beta_{4,3} & \beta_{4,4} & \beta_{4,5} & \beta_{4,6} & \beta_{4,7} \\ \beta_{5,1} & \beta_{5,2} & \beta_{5,3} & \beta_{5,4} & \beta_{5,5} & \beta_{5,6} & \beta_{5,7} \\ \beta_{6,1} & \beta_{6,2} & \beta_{6,3} & \beta_{6,4} & \beta_{6,5} & \beta_{6,6} & \beta_{6,7} \\ \beta_{7,1} & \beta_{7,2} & \beta_{7,3} & \beta_{7,4} & \beta_{7,5} & \beta_{7,6} & \beta_{7,7} \end{bmatrix} \begin{bmatrix} \Delta Y_{t-1} \\ \Delta IR_{t-1} \\ \Delta S_{t-1} \\ \Delta \Pi_{t-1} \\ \Delta YF1_{t-1} \\ \Delta YF2_{t-1} \\ \Delta YF3_{t-1} \end{bmatrix} + \begin{bmatrix} \beta_{1,8} \\ \beta_{2,8} \\ \beta_{3,8} \\ \beta_{4,8} \\ \beta_{5,8} \\ \beta_{6,8} \\ \beta_{7,8} \end{bmatrix} [\Delta Y_t^{EU}] + \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \varepsilon_{3t} \\ \varepsilon_{4t} \\ \varepsilon_{5t} \\ \varepsilon_{6t} \\ \varepsilon_{7t} \end{bmatrix}. \quad (7)$$

Estimation of the above equation yields the following results:

$$\Delta Y_t = 0.174 + \mathbf{0.544 \Delta Y_{t-1}} - 0.001 \Delta IR_{t-1} + \mathbf{0.084 \Delta S_{t-1}} + 0.092 \Pi_{t-1} + 0.003 \Delta YF1_{t-1} \quad (8)$$

[0.968] [7.032] [-0.501] [2.644] [0.686] [1.594]

$$-9.359e^{-5} \Delta YF2_{t-1} + 0.0002 \Delta YF3_{t-1} + \mathbf{0.707 \Delta Y_t^{EU}}$$

[-0.522] [1.123] [8.083]

R^2=0.95, Adjusted R^2= 93,

$$\Delta IR_t = -3.943 - 0.850 \Delta Y_{t-1} + \mathbf{0.949 \Delta IR_{t-1}} + 0.735 \Delta S_{t-1} + \mathbf{6.165 \Pi_{t-1}} - 0.004 \Delta YF1_{t-1} \quad (9)$$

[-0.818] [-0.423] [15.922] [0.867] [1.713] [-0.840]

$$-0.001 \Delta YF2_{t-1} + 0.004 \Delta YF3_{t-1} + \mathbf{5.749 \Delta Y_t^{EU}}$$

[-0.183] [0.865] [2.488]

R^2=0.93, Adjusted R^2= 90,

$$\Delta S_t = 0.242 + \mathbf{0.537\Delta Y_{t-1}} + 0.003\Delta IR_{t-1} + \mathbf{0.722\Delta S_{t-1}} - 0.461\Pi_{t-1} - 0.001\Delta YF1_{t-1} \quad (10)$$

[0.373] [1.973] [0.342] [5.826] [-0.951] [-1.625]

$$-0.001\Delta YF2_{t-1} - 0.001\Delta YF3_{t-1} - \mathbf{1.156\Delta Y_t^{EU}}$$

[-1.288] [-0.966] [-3.718]

R^2=0.78, Adjusted R^2= 71,

$$\Delta\Pi_t = 0.097 - \mathbf{0.091\Delta Y_{t-1}} + 0.001\Delta IR_{t-1} - 0.002\Delta S_{t-1} + \mathbf{0.788\Pi_{t-1}} \quad (11)$$

[0.755] [-1.728] [0.660] [-0.097] [7.944]

$$+4.222e^{-5}\Delta YF1_{t-1} + 8.320e^{-5}\Delta YF2_{t-1} - 0.0002\Delta YF3_{t-1} + \mathbf{0.266\Delta Y_t^{EU}}$$

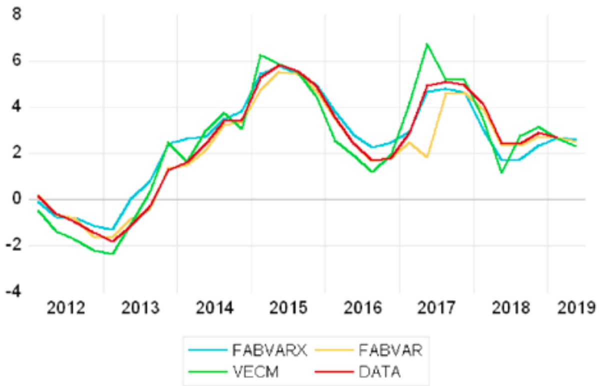
[0.322] [0.660] [-1.783] [4.406]

R^2=0.86, Adjusted R^2= 80.

The R-squared but also adjusted R-squared informs about a much higher proportion of variance for the dependent variable Y_t to be explained by data when estimated with the FABVARX model relatively to the two previous models. The introduction of the external forecast for the EU GDP proves to be beneficial. Graph 1 summarizes the findings from the three above described models.

Graph 1: GDP Forecast

YoY growth rate, in %



Source: Author's calculations.

3.2 Consumption Models

3.2.1 VAR Model

The first applied model is a simple vector autoregressive model which estimates the relationship between two variables, consumption C_t and wages W_t. The theoretical but also empirical relationship between these variables is indisputable. The model in its general form can be written as:

$$\begin{bmatrix}\Delta C_t \\ \Delta W_t\end{bmatrix} = \begin{bmatrix}\beta_{1,0} \\ \beta_{2,0}\end{bmatrix} + \begin{bmatrix}\beta_{1,1} & \beta_{1,2} \\ \beta_{2,1} & \beta_{2,2}\end{bmatrix}\begin{bmatrix}\Delta C_{t-1} \\ \Delta W_{t-1}\end{bmatrix} + \begin{bmatrix}\varepsilon_{1t} \\ \varepsilon_{2t}\end{bmatrix}. \quad (12)$$

Estimation of the above model results in the subsequent set of equations:

$$\Delta C_t = -0.0464 + \mathbf{0.802\Delta C_{t-1}} + \mathbf{0.109\Delta W_{t-1}} \quad (13)$$

[-0.250] [1.956] [9.847]

R^2=0.83,

$$\Delta W_t = 0.488 + 0.240\Delta C_{t-1} + \mathbf{0.738\Delta W_{t-1}} \quad (14)$$

[1.385] [1.557] [6.993]

R^2=0.71.

The R-squared for the VAR model is relatively high over 70 percent.

3.2.2 FAVAR Model

The second selected model augments the simple vector autoregressive model for the inclusion of factors $CF1_t$, $CF2_t$ and $CF3_t$. These factors summarize information about a common trend among variables such as GDP, investment, export, import, government consumption, labour demand, labour supply, M2 and capital. The model will benefit from the extra information regarding the evolution of multiple series. The general representation of the model is following:

$$\begin{bmatrix} \Delta C_t \\ \Delta W_t \\ \Delta CF1_t \\ \Delta CF2_t \\ \Delta CF3_t \end{bmatrix} = \begin{bmatrix} \beta_{1,0} \\ \beta_{2,0} \\ \beta_{3,0} \\ \beta_{4,0} \\ \beta_{5,0} \end{bmatrix} + \begin{bmatrix} \beta_{1,1} & \beta_{1,2} & \beta_{1,3} & \beta_{1,4} & \beta_{1,5} \\ \beta_{2,1} & \beta_{2,2} & \beta_{2,3} & \beta_{2,4} & \beta_{2,5} \\ \beta_{3,1} & \beta_{3,2} & \beta_{3,3} & \beta_{3,4} & \beta_{3,5} \\ \beta_{4,1} & \beta_{4,2} & \beta_{4,3} & \beta_{4,4} & \beta_{4,5} \\ \beta_{5,1} & \beta_{5,2} & \beta_{5,3} & \beta_{5,4} & \beta_{5,5} \end{bmatrix} \begin{bmatrix} \Delta C_{t-1} \\ \Delta W_{t-1} \\ \Delta CF1_{t-1} \\ \Delta CF2_{t-1} \\ \Delta CF3_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \varepsilon_{3t} \\ \varepsilon_{4t} \\ \varepsilon_{5t} \end{bmatrix}. \quad (15)$$

The estimation outcome is visible on these equations:

$$\Delta C_t = -0.0616 + \mathbf{0.855\Delta C_{t-1}} + 0.064\Delta W_{t-1} + \mathbf{0.001\Delta CF1_{t-1}} \quad (16)$$

[-0.342] [10.792] [1.103] [2.660]

$$+0.0001CF2_{t-1} + 7.043e^{-5}\Delta CF3_{t-1}$$

[0.666] [1.012]

R^2=0.86,

$$\Delta W_t = 0.449 + 0.210\Delta C_{t-1} + \mathbf{0.766\Delta W_{t-1}} - 0.0002\Delta CF1_{t-1} \quad (17)$$

[1.195] [1.273] [6.343] [0.507]

$$-0.0001CF2_{t-1} + 8.572e^{-5}\Delta CF3_{t-1}$$

[-0.306] [0.590]

R^2=0.72.

Relatively high R-squared informs about a satisfactory explanation of the time-series variation by the model.

3.2.3 FABVAR Model

The last selected model for forecasting consumption variable is the FABVAR model. This model combines advantages of FAVAR models with benefits of Bayesian estimation. Again, three factors which explain more than 50 percent variation of the data were chosen, specifically $CF1_t$, $CF2_t$ and $CF3_t$. For the Bayesian estimation, we assume Minnesota priors. The model can be represented as:

$$\begin{bmatrix} \Delta C_t \\ \Delta W_t \\ \Delta CF1_t \\ \Delta CF2_t \\ \Delta CF3_t \end{bmatrix} = \begin{bmatrix} \beta_{1,0} \\ \beta_{2,0} \\ \beta_{3,0} \\ \beta_{4,0} \\ \beta_{5,0} \end{bmatrix} + \begin{bmatrix} \beta_{1,1} & \beta_{1,2} & \beta_{1,3} & \beta_{1,4} & \beta_{1,5} \\ \beta_{2,1} & \beta_{2,2} & \beta_{2,3} & \beta_{2,4} & \beta_{2,5} \\ \beta_{3,1} & \beta_{3,2} & \beta_{3,3} & \beta_{3,4} & \beta_{3,5} \\ \beta_{4,1} & \beta_{4,2} & \beta_{4,3} & \beta_{4,4} & \beta_{4,5} \\ \beta_{5,1} & \beta_{5,2} & \beta_{5,3} & \beta_{5,4} & \beta_{5,5} \end{bmatrix} \begin{bmatrix} \Delta C_{t-1} \\ \Delta W_{t-1} \\ \Delta CF1_{t-1} \\ \Delta CF2_{t-1} \\ \Delta CF3_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \varepsilon_{3t} \\ \varepsilon_{4t} \\ \varepsilon_{5t} \end{bmatrix}. \quad (18)$$

Estimation of the above equations results in:

$$\Delta C_t = -0.036 + \mathbf{0.855\Delta C_{t-1}} + 0.063\Delta W_{t-1} + \mathbf{0.0004\Delta CF1_{t-1}} \quad (19)$$

[-0.187] [10.379] [1.119] [2.124]

$$+8.221e^{-5}CF2_{t-1} + 5.009e^{-5}\Delta CF3_{t-1}$$

[0.503] [0.770]

R^2=0.86,

$$\Delta W_t = 0.444 + 0.187\Delta C_{t-1} + \mathbf{0.779\Delta W_{t-1}} - 0.0002\Delta CF1_{t-1} \quad (20)$$

[1.218] [1.237] [7.052] [-0.493]

$$-7.918e^{-5}CF2_{t-1} + 6.372e^{-5}\Delta CF3_{t-1}\ .$$

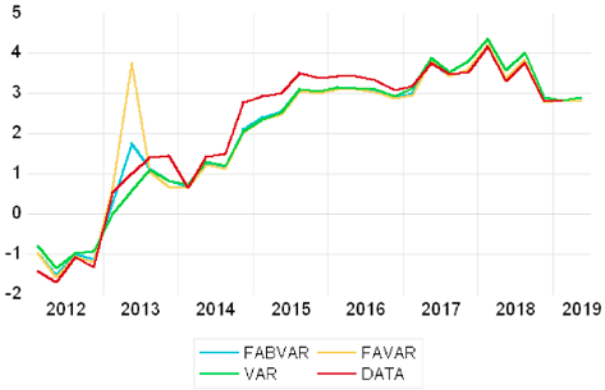
[-0.260] [0.524]

R^2=0.72.

The R-squared values are comparable to those from the FAVAR model. The next graph illustrates the one year ahead in-sample forecasting performance of the three models.

Graph 2: Consumption Forecast

YoY growth rate, in %



Source: Author's calculations.

3.3 Investment Models

3.3.1 VAR Model

The first estimated model for analysing the investment variable is a vector autoregressive model. This rather simple model will include only one lag. The following small number of unknown parameters predetermines the potential of the model to serve well also for out-of-sample forecasting. The model tests endogenous relationship between GDP and investment:

$$\begin{bmatrix} \Delta I_t \\ \Delta Y_t \end{bmatrix} = \begin{bmatrix} \beta_{1,0} \\ \beta_{2,0} \end{bmatrix} + \begin{bmatrix} \beta_{1,1} & \beta_{1,2} \\ \beta_{2,1} & \beta_{2,2} \end{bmatrix} \begin{bmatrix} \Delta I_{t-1} \\ \Delta Y_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix}. \quad (21)$$

Estimation of the general form of the model yields:

$$\Delta I_t = -1.360 + \mathbf{0.553}\Delta \boldsymbol{I_{t-1}} + 0.922\Delta Y_{t-1} \quad (22)$$

[-1.128] [3.099] [1.603]

R^2=0.67,

$$\Delta Y_t = 0.181 + 0.003\Delta I_{t-1} + \mathbf{0.862}\Delta \boldsymbol{Y_{t-1}}. \quad (23)$$

[0.607] [0.078] [6.065]

R^2=0.79.

The estimation results show around 70 percent of the data to be explained by the model. However, analysis of individual equations reveals each variable to depend significantly only on its own lag.

3.3.2 FABVAR Model

The second selected model utilized for forecasting the investment variable is a factor augmented Bayesian vector autoregressive model. This type of a model shrinks number of unknown parameters by first detecting trends among chosen variables such as consumption, government consumption, export, import, labour demand, labour supply, M2, wage, price of investment and capital. These trends are summarized by three principal components $IF1_t$, $IF2_t$ and $IF3_t$. Next, an introduction of new information in the form of a priory beliefs about parameter values for Bayesian estimation further reduces uncertainty surrounding the parameters' values. As well as in the previous models, Minnesota priors are assumed to be the most appropriate priors for the parameters in the model. A general representation of the model is:

$$\begin{bmatrix} \Delta I_t \\ \Delta Y_t \\ \Delta IF1_t \\ \Delta IF2_t \\ \Delta IF3_t \end{bmatrix} = \begin{bmatrix} \beta_{1,0} \\ \beta_{2,0} \\ \beta_{3,0} \\ \beta_{4,0} \\ \beta_{5,0} \end{bmatrix} + \begin{bmatrix} \beta_{1,1} & \beta_{1,2} & \beta_{1,3} & \beta_{1,4} & \beta_{1,5} \\ \beta_{2,1} & \beta_{2,2} & \beta_{2,3} & \beta_{2,4} & \beta_{2,5} \\ \beta_{3,1} & \beta_{3,2} & \beta_{3,3} & \beta_{3,4} & \beta_{3,5} \\ \beta_{4,1} & \beta_{4,2} & \beta_{4,3} & \beta_{4,4} & \beta_{4,5} \\ \beta_{5,1} & \beta_{5,2} & \beta_{5,3} & \beta_{5,4} & \beta_{5,5} \end{bmatrix} \begin{bmatrix} \Delta I_{t-1} \\ \Delta Y_{t-1} \\ \Delta IF1_{t-1} \\ \Delta IF2_{t-1} \\ \Delta IF3_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \varepsilon_{3t} \\ \varepsilon_{4t} \\ \varepsilon_{5t} \end{bmatrix}. \quad (24)$$

Outcome of the model estimation results in:

$$\Delta I_t = -0.773 + \mathbf{0.661\Delta I_{t-1}} + 0.571\Delta Y_{t-1} + 0.001\Delta IF1_{t-1} \quad (25)$$

[-0.655] [3.948] [1.072] [0.132]

$$+0.0005IF2_{t-1} + 0.002\Delta IF3_{t-1}$$

[0.362] [0.260]

R^2=0.67,

$$\Delta Y_t = 0.171 - 0.006\Delta I_{t-1} + \mathbf{0.894\Delta Y_{t-1}} - 0.001\Delta IF1_{t-1} \quad (26)$$

[0.605] [-0.155] [7.083] [-0.739]

$$-7.338e^{-5}IF2_{t-1} + 0.0001\Delta IF3_{t-1}.$$

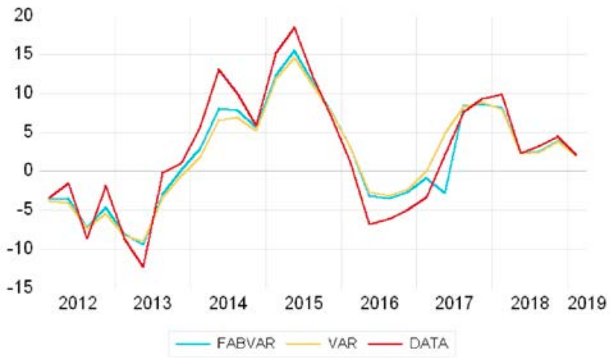
[-0.245] [0.860]

R^2=0.80.

The FABVAR model benefits from relatively higher R-squared compared to the simple VAR model. The final graph proves high accuracy of the out-of-sample forecasting ability connected with the previous models.

Graph 3: Investment Forecast

YoY growth rate, in %



Source: Author's calculations.

3.4 Export Models

This sub-section focuses on the estimation of the export variable with two models.

3.4.1 BVARX Model

The dynamics of the Czech economy as a small open economy significantly depends on the economic evolution of its foreign partners. An advantage of the BVARX model is its ability to control for such exogenous evolution of a variable, in this case the EU GDP. Other important variable for forecasting the export variable is undoubtedly exchange rate. Finally, the Bayesian estimation of the model with Minnesota priors provides the variables with an assumption of following a random walk at least into some degree. The model is:

$$\begin{bmatrix}\Delta X_t \\ \Delta S_t\end{bmatrix} = \begin{bmatrix}\beta_{1,0} \\ \beta_{2,0}\end{bmatrix} + \begin{bmatrix}\beta_{1,1} & \beta_{1,2} \\ \beta_{2,1} & \beta_{2,2}\end{bmatrix}\begin{bmatrix}\Delta X_{t-1} \\ \Delta S_{t-1}\end{bmatrix} + \begin{bmatrix}\beta_{1,3} \\ \beta_{2,3}\end{bmatrix}[\Delta Y_t^{EU}] + \begin{bmatrix}\varepsilon_{1t} \\ \varepsilon_{2t}\end{bmatrix}. \quad (27)$$

Bayesian estimation of the above model results in the following equations:

$$\Delta X_t = -1.149 + \mathbf{0.571\Delta X_{t-1}} + \mathbf{0.335\Delta S_{t-1}} + \mathbf{1.454\Delta Y_t^{EU}} \quad (28)$$

[1.656] [4.224] [2.951] [3.369]

R^2=0.81,

$$\Delta S_t = 0.453 - 0.011\Delta \boldsymbol{X_{t-1}} + \mathbf{0.698\Delta S_{t-1}} - \mathbf{0.724\Delta Y_t^{EU}} \quad (29)$$

[0.841] [-0.108] [7.437] [-2.226]

R^2=0.70.

The BVARX model seems to be an appropriate model for estimating the export variable. The R-squared is relatively high above 70 percent plus all selected variables appear as statistically significant. Foreign GDP positively affects Czech export as well as CZK/EUR depreciation.

3.4.2 FABVARX Model

A factor augment Bayesian vector autoregressive model with an exogenous variable is chosen to serve as a base for comparison to the previous BVARX model. In contrast to the BVARX model, the FABVARX model enables an inclusion of additional variables without significantly reducing an out-of-sample forecasting accuracy.

GDP, consumption, investment, import, government consumption, labour demand, labour supply, wage, price of export, M2 and capital are all variables utilized to determine three principal components $XF1_t$, $XF2_t$ and $XF3_t$. The FABVARX model can be written in a following way:

$$\begin{bmatrix} \Delta X_t \\ \Delta S_t \\ \Delta XF1_t \\ \Delta XF2_t \\ \Delta XF3_t \end{bmatrix} = \begin{bmatrix} \beta_{1,0} \\ \beta_{2,0} \\ \beta_{3,0} \\ \beta_{4,0} \\ \beta_{5,0} \end{bmatrix} + \begin{bmatrix} \beta_{1,1} & \beta_{1,2} & \beta_{1,3} & \beta_{1,4} & \beta_{1,5} \\ \beta_{2,1} & \beta_{2,2} & \beta_{2,3} & \beta_{2,4} & \beta_{2,5} \\ \beta_{3,1} & \beta_{3,2} & \beta_{3,3} & \beta_{3,4} & \beta_{3,5} \\ \beta_{4,1} & \beta_{4,2} & \beta_{4,3} & \beta_{4,4} & \beta_{4,5} \\ \beta_{5,1} & \beta_{5,2} & \beta_{5,3} & \beta_{5,4} & \beta_{5,5} \end{bmatrix} \begin{bmatrix} \Delta X_{t-1} \\ \Delta S_{t-1} \\ \Delta XF1_{t-1} \\ \Delta XF2_{t-1} \\ \Delta XF3_{t-1} \end{bmatrix} + \begin{bmatrix} \beta_{1,8} \\ \beta_{2,8} \\ \beta_{3,8} \\ \beta_{4,8} \\ \beta_{5,8} \end{bmatrix} [\Delta Y_t^{EU}] + \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \varepsilon_{3t} \\ \varepsilon_{4t} \\ \varepsilon_{5t} \end{bmatrix}. \tag{30}$$

The estimated equations appear as:

$$\Delta X_t = 1.075 + \mathbf{0.561\Delta X_{t-1}} + \mathbf{0.365\Delta S_{t-1}} + 0.001\Delta XF1_{t-1} \tag{31}$$

[1.539] [4.063] [2.957] [0.663]

$$-0.0003XF2_{t-1} - 4.306e^{-5}\Delta XF3_{t-1} + \mathbf{1.510\Delta Y_t^{EU}}$$

[-0.536] [-0.289] [3.433]

R^2=0.82,

$$\Delta S_t = 0.557 + 0.024\Delta X_{t-1} + 0.678\Delta S_{t-1} - 0.002\Delta XF1_{t-1} \tag{32}$$

[1.025] [0.239] [6.566] [-1.648]

$$-0.0003XF2_{t-1} + 0.0001\Delta XF3_{t-1} - \mathbf{0.829\Delta Y_t^{EU}}$$

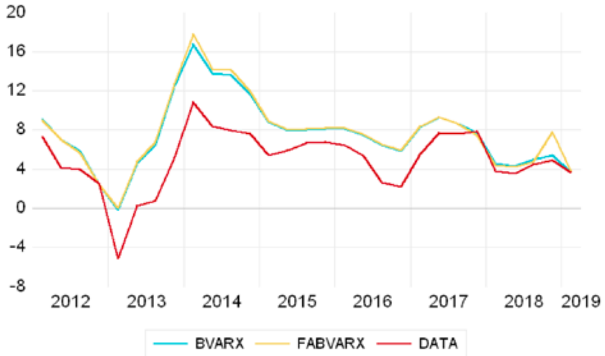
[-0.584] [0.993] [-2.507]

R^2=0.74.

The inclusion of the factors increases the R-squared of the model while keeping the same variable statistically significant for the export equation as with the BVARX model. The resulting out-of-sample forecast proves the FABVARX model to yield a more optimistic forecast.

Graph 4: Export Forecast

YoY growth rate, in %



Source: Author's calculations.

3.5 Import Models

Alike the estimation of the export variable, we find the same forms of models to yield the best results for the import variable.

3.5.1 BVARX Model

A Bayesian vector autoregressive model with an exogenous variable is utilized for estimating the import variable assimilates the BVARX model estimated for the export variable. Similarly, this model includes the exchange rate CZK/EUR and the EU GDP. The model appears as:

$$\begin{bmatrix}\Delta M_t \\ \Delta S_t\end{bmatrix} = \begin{bmatrix}\beta_{1,0} \\ \beta_{2,0}\end{bmatrix} + \begin{bmatrix}\beta_{1,1} & \beta_{1,2} \\ \beta_{2,1} & \beta_{2,2}\end{bmatrix}\begin{bmatrix}\Delta M_{t-1} \\ \Delta S_{t-1}\end{bmatrix} + \begin{bmatrix}\beta_{1,3} \\ \beta_{2,3}\end{bmatrix}[\Delta Y_t^{EU}] + \begin{bmatrix}\varepsilon_{1t} \\ \varepsilon_{2t}\end{bmatrix}. \tag{33}$$

Estimation of the model results in the subsequent set of equations:

$$\Delta M_t = 0.933 + \mathbf{0.55}\Delta \boldsymbol{M_{t-1}} + \mathbf{0.319}\Delta \boldsymbol{S_{t-1}} + \mathbf{1.653}\Delta \boldsymbol{Y_t^{EU}} \tag{34}$$

[1.488] [4.183] [2.842] [3.743]

R^2=0.82,

$$\Delta S_t = 0.380 + 0.018\Delta M_{t-1} + \mathbf{0.707}\Delta \boldsymbol{S_{t-1}} - \mathbf{0.799}\Delta \boldsymbol{Y_t^{EU}} \tag{35}$$

[0.758] [0.198] [7.760] [-2.443]

R^2=0.70.

The R-squared proves this model to explain a significant portion of the import variations. The positive sign in front of the exchange rate for the import variable might originate in the high import intensity of the Czech export. As the export is not present in the equations, we do not control for the direct impact of the Czech export on the import variable. Therefore, exchange rate depreciation leading to export increases might positively affect import. By other words, firms exporting goods are in need of more inputs imported from abroad.

3.5.2 FABVARX Model

A factor augmented Bayesian vector autoregressive model with an exogenous variable includes a similar set of variables as the one utilized for forecasting the export variable. Selected variables for estimating factors are GDP, investment, import, consumption, government consumption, labour demand, labour supply, M2, capital and price of import. Next to factors, another endogenous variable in the model is exchange rate CZK/EUR. The model also includes exogenous variable, the EU GDP, which proves to be a significant variable affecting dynamics of the Czech economy. The final model takes on the following form:

$$\begin{bmatrix}\Delta M_t \\ \Delta S_t \\ \Delta MF1_t \\ \Delta MF2_t \\ \Delta MF3_t\end{bmatrix} = \begin{bmatrix}\beta_{1,0} \\ \beta_{2,0} \\ \beta_{3,0} \\ \beta_{4,0} \\ \beta_{5,0}\end{bmatrix} + \begin{bmatrix}\beta_{1,1} & \beta_{1,2} & \beta_{1,3} & \beta_{1,4} & \beta_{1,5} \\ \beta_{2,1} & \beta_{2,2} & \beta_{2,3} & \beta_{2,4} & \beta_{2,5} \\ \beta_{3,1} & \beta_{3,2} & \beta_{3,3} & \beta_{3,4} & \beta_{3,5} \\ \beta_{4,1} & \beta_{4,2} & \beta_{4,3} & \beta_{4,4} & \beta_{4,5} \\ \beta_{5,1} & \beta_{5,2} & \beta_{5,3} & \beta_{5,4} & \beta_{5,5}\end{bmatrix}\begin{bmatrix}\Delta M_{t-1} \\ \Delta S_{t-1} \\ \Delta MF1_{t-1} \\ \Delta MF2_{t-1} \\ \Delta MF3_{t-1}\end{bmatrix} + \begin{bmatrix}\beta_{1,8} \\ \beta_{2,8} \\ \beta_{3,8} \\ \beta_{4,8} \\ \beta_{5,8}\end{bmatrix}[\Delta Y_t^{EU}] + \begin{bmatrix}\varepsilon_{1t} \\ \varepsilon_{2t} \\ \varepsilon_{3t} \\ \varepsilon_{4t} \\ \varepsilon_{5t}\end{bmatrix}. \tag{36}$$

Substituting the general parameters for the estimated ones can be written as:

$$\Delta M_t = 0.933 + \mathbf{0.514}\Delta \boldsymbol{M_{t-1}} + \mathbf{0.324}\Delta \boldsymbol{S_{t-1}} + 0.001\Delta MF1_{t-1} \tag{37}$$

[1.415] [3.928] [2.746] [0.400]

$$+0.0005MF2_{t-1} + 0.0003\Delta MF3_{t-1} + \mathbf{1.753}\Delta \boldsymbol{Y_t^{EU}}$$

[0.495] [0.558] [3.814]

R^2=0.82,

$$\Delta S_t = 0.529 + 0.045\Delta M_{t-1} + \mathbf{0.666}\Delta \boldsymbol{S_{t-1}} - 0.003\Delta MF1_{t-1} \tag{38}$$

[1.042] [0.483] [6.890] [-1.566]

$$+0.0002MF2_{t-1} - 0.0004\Delta MF3_{t-1} - \mathbf{0.917}\Delta \boldsymbol{Y_t^{EU}}$$

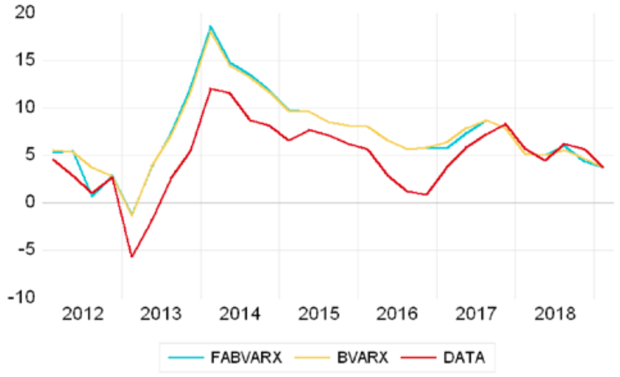
[0.263] [-0.994] [-2.703]

R^2=0.73.

The results assimilate the ones from the previous model. The R-squared is slightly higher for the FABVARX model. The next graph summarizes the results for the out-of-sample forecast for both models.

Graph 5: Import Forecast

YoY growth rate, in %



Source: Author's calculations.

3.6 Inflation Models

3.6.1 BVARX

A complex BVARX is utilized for the econometric forecast of the inflation variable. This model benefits from the inclusion of the exogenous forecast for the EU GDP. The GDP of the EU significantly affects the Czech economy and in turn its price level. Other included variable important for estimating the inflation evolution is the nominal interest rate and the exchange rate. Finally, we control for the evolution of the Czech production by incorporating GDP variable into the model. The model is estimated with a Bayesian method.

$$\begin{bmatrix} \Pi_t \\ \Delta Y_t \\ \Delta IR_t \\ \Delta S_t \end{bmatrix} = \begin{bmatrix} \beta_{1,0} \\ \beta_{2,0} \end{bmatrix} + \begin{bmatrix} \beta_{1,1} & \beta_{1,2} \\ \beta_{2,1} & \beta_{2,2} \end{bmatrix} \begin{bmatrix} \Pi_{t-1} \\ \Delta Y_{t-1} \\ \Delta IR_{t-1} \\ \Delta S_{t-1} \end{bmatrix} + \begin{bmatrix} \beta_{1,3} \\ \beta_{2,3} \\ \beta_{3,3} \\ \beta_{4,3} \end{bmatrix} [\Delta Y_t^{EU}] + \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \varepsilon_{3t} \\ \varepsilon_{4t} \end{bmatrix}. \tag{39}$$

The above general formulation of the model yields following results:

$$\Delta\Pi_t = 0.126 - 0.087\Delta Y_{t-1} + 0.001\Delta IR_{t-1} + 0.012\Delta S_{t-1} \tag{40}$$

[1.062] [-1.667] [0.792] [0.639]

$$+\mathbf{0.813\Pi_{t-1}} + \mathbf{0.260\Delta Y_t^{EU}}$$

[8.564] [4.316]

R^2=0.84,

$$\Delta Y_t = 0.150 + \mathbf{0.535\Delta Y_{t-1}} - 0.002\Delta IR_{t-1} + \mathbf{0.060\Delta S_{t-1}} \tag{41}$$

[0.896] [7.042] [-0.751] [2.163]

$$+0.117\Pi_{t-1} + \mathbf{0.707\Delta Y_t^{EU}}$$

[0.907] [8.122]

R^2=0.94,

$$\Delta IR_t = -4.683 - 0.864\Delta \mathbf{Y_{t-1}} + \mathbf{0.949\Delta IR_{t-1}} + 0.614\Delta S_{t-1} \tag{42}$$

[-1.042] [-0.438] [-16.053] [0.831]

$$+5.360\Pi_{t-1} + \mathbf{5.921\Delta Y_t^{EU}}$$

[1.549] [2.574]

R^2=0.92,

$$\Delta S_t = 0.603 + \mathbf{0.479}\Delta \boldsymbol{Y}_{\boldsymbol{t-1}} + 0.004\Delta IR_{t-1} + \mathbf{0.721}\Delta \boldsymbol{S}_{\boldsymbol{t-1}} \tag{43}$$

[0.997] [1.799] [0.548] [6.824]

$$-0.736\Pi_{t-1} - \mathbf{1.104}\Delta \boldsymbol{Y}_{\boldsymbol{t}}^{\boldsymbol{EU}}$$

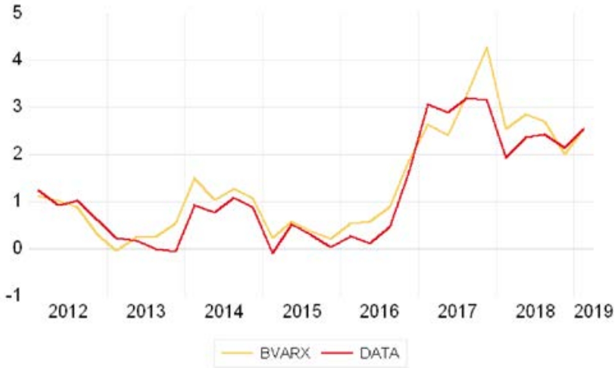
[-1.583] [-3.571]

R^2=0.73.

The one period ahead out-of-sample forecast for the BVARX model is illustrated on the graph.

Graph 6: Inflation Forecast

In %



Source: Author's calculations.

3.7 Wage Models

3.7.1 VAR Model

A vector autoregressive model is a simplest model used for estimating the wage variable. The model includes besides the wage variable, the consumption variable. The strong link between wages and consumption was already visible in the models designed for forecasting consumption.

The model takes on the following form:

$$\begin{bmatrix}\Delta W_t \\ \Delta C_t\end{bmatrix} = \begin{bmatrix}\beta_{1,0} \\ \beta_{2,0}\end{bmatrix} + \begin{bmatrix}\beta_{1,1} & \beta_{1,2} \\ \beta_{2,1} & \beta_{2,2}\end{bmatrix}\begin{bmatrix}\Delta W_{t-1} \\ \Delta C_{t-1}\end{bmatrix} + \begin{bmatrix}\varepsilon_{1t} \\ \varepsilon_{2t}\end{bmatrix}. \tag{44}$$

Estimation of the above model provides following set of equations:

$$\Delta C_t = -0.0464 + \mathbf{0.802}\Delta \boldsymbol{C}_{\boldsymbol{t-1}} + \mathbf{0.109}\Delta \boldsymbol{W}_{\boldsymbol{t-1}} \tag{45}$$

[-0.250] [1.956] [9.847]

R^2=0.83,

$$\Delta W_t = 0.488 + 0.240\Delta C_{t-1} + \mathbf{0.738}\Delta \boldsymbol{W}_{\boldsymbol{t-1}} \tag{46}$$

[1.385] [1.557] [6.993]

R^2=0.71.

Wage appears to be a significant determinant of the consumption variance.

3.7.2 FAVAR Model

The next model is also a vector autoregressive model but augmented for three factors. The factors combine trend among multiple variables such as GDP, government consumption, investment, labour demand, labour supply, import, export, M2 and capital. The selected factors explain more than fifty percent of the data variability.

$$\begin{bmatrix}\Delta W_t\\ \Delta C_t\\ \Delta CF1_t\\ \Delta CF2_t\\ \Delta CF3_t\end{bmatrix}=\begin{bmatrix}\beta_{1,0}\\ \beta_{2,0}\\ \beta_{3,0}\\ \beta_{4,0}\\ \beta_{5,0}\end{bmatrix}+\begin{bmatrix}\beta_{1,1} & \beta_{1,2} & \beta_{1,3} & \beta_{1,4} & \beta_{1,5}\\ \beta_{2,1} & \beta_{2,2} & \beta_{2,3} & \beta_{2,4} & \beta_{2,5}\\ \beta_{3,1} & \beta_{3,2} & \beta_{3,3} & \beta_{3,4} & \beta_{3,5}\\ \beta_{4,1} & \beta_{4,2} & \beta_{4,3} & \beta_{4,4} & \beta_{4,5}\\ \beta_{5,1} & \beta_{5,2} & \beta_{5,3} & \beta_{5,4} & \beta_{5,5}\end{bmatrix}\begin{bmatrix}\Delta W_{t-1}\\ \Delta C_{t-1}\\ \Delta CF1_{t-1}\\ \Delta CF2_{t-1}\\ \Delta CF3_{t-1}\end{bmatrix}+\begin{bmatrix}\varepsilon_{1t}\\ \varepsilon_{2t}\\ \varepsilon_{3t}\\ \varepsilon_{4t}\\ \varepsilon_{5t}\end{bmatrix}. \quad (47)$$

An estimation of the above general form of the model provides following result:

$$\Delta C_t = -0.0616 + \mathbf{0.855}\Delta \boldsymbol{C_{t-1}} + 0.064\Delta W_{t-1} + \mathbf{0.001}\Delta \boldsymbol{CF1_{t-1}} \quad (48)$$

[-0.342] [10.792] [1.103] [2.660]

$$+0.0001CF2_{t-1} + 7.043e^{-5}\Delta CF3_{t-1}$$

[0.666] [1.012]

R^2=0.86,

$$\Delta W_t = 0.449 + 0.210\Delta C_{t-1} + \mathbf{0.766}\Delta \boldsymbol{W_{t-1}} - 0.0002\Delta CF1_{t-1} \quad (49)$$

[1.195] [1.273] [6.343] [0.507]

$$-0.0001CF2_{t-1} + 8.572e^{-5}\Delta CF3_{t-1}\,.$$

[-0.306] [0.590]

R^2=0.72.

The existence of factors along with the statistical significance of the first principal component lead to a reduction of the statistical significance for the wage variable in the consumption equation.

3.7.3 FABVAR Model

Bayesian estimation of the previous FAVAR model:

$$\begin{bmatrix}\Delta W_t\\ \Delta C_t\\ \Delta CF1_t\\ \Delta CF2_t\\ \Delta CF3_t\end{bmatrix}=\begin{bmatrix}\beta_{1,0}\\ \beta_{2,0}\\ \beta_{3,0}\\ \beta_{4,0}\\ \beta_{5,0}\end{bmatrix}+\begin{bmatrix}\beta_{1,1} & \beta_{1,2} & \beta_{1,3} & \beta_{1,4} & \beta_{1,5}\\ \beta_{2,1} & \beta_{2,2} & \beta_{2,3} & \beta_{2,4} & \beta_{2,5}\\ \beta_{3,1} & \beta_{3,2} & \beta_{3,3} & \beta_{3,4} & \beta_{3,5}\\ \beta_{4,1} & \beta_{4,2} & \beta_{4,3} & \beta_{4,4} & \beta_{4,5}\\ \beta_{5,1} & \beta_{5,2} & \beta_{5,3} & \beta_{5,4} & \beta_{5,5}\end{bmatrix}\begin{bmatrix}\Delta W_{t-1}\\ \Delta C_{t-1}\\ \Delta CF1_{t-1}\\ \Delta CF2_{t-1}\\ \Delta CF3_{t-1}\end{bmatrix}+\begin{bmatrix}\varepsilon_{1t}\\ \varepsilon_{2t}\\ \varepsilon_{3t}\\ \varepsilon_{4t}\\ \varepsilon_{5t}\end{bmatrix} \quad (50)$$

results in:

$$\Delta C_t = -0.036 + \mathbf{0.855}\Delta \boldsymbol{C_{t-1}} + 0.063\Delta W_{t-1} + \mathbf{0.0004}\Delta \boldsymbol{CF1_{t-1}} \quad (51)$$

[-0.187] [10.379] [1.119] [2.124]

$$+8.221e^{-5}CF2_{t-1} + 5.009e^{-5}\Delta CF3_{t-1}$$

[0.503] [0.770]

R^2=0.86,

$$\Delta W_t = 0.444 + 0.187\Delta C_{t-1} + \mathbf{0.779}\Delta \boldsymbol{W_{t-1}} - 0.0002\Delta CF1_{t-1} \quad (52)$$

[1.218] [1.237] [7.052] [-0.493]

$$-7.918e^{-5}CF2_{t-1} + 6.372e^{-5}\Delta CF3_{t-1}$$

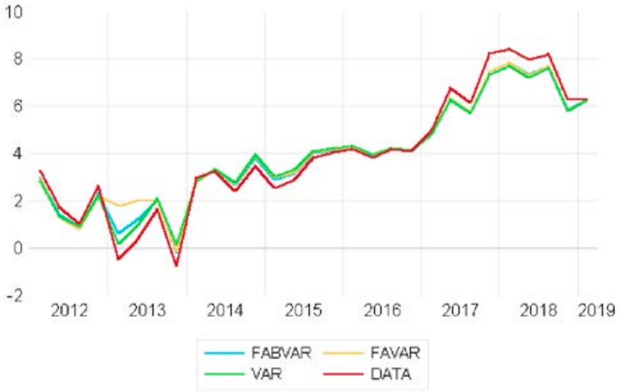
[-0.260] [0.524]

R^2=0.72.

The above results are similar to those from the FAVAR model. The one period ahead out-of-sample forecast of all the models can be summarized by the below graph.

Graph 7: Wage Forecast

YoY growth rate, in %



Source: Author's calculations.

3.8 Exchange Rate Models

The selected models for Exchange rate are the same models used for estimating the import and the export variables. These models have proven to provide the most accurate forecasts for the Exchange rate.

3.8.1 BVARX-M Model

The Bayesian vector autoregressive model with the exogenous variable used for modelling the import variable is formulated as:

$$\begin{bmatrix} \Delta S_t \\ \Delta M_t \end{bmatrix} = \begin{bmatrix} \beta_{1,0} \\ \beta_{2,0} \end{bmatrix} + \begin{bmatrix} \beta_{1,1} & \beta_{1,2} \\ \beta_{2,1} & \beta_{2,2} \end{bmatrix} \begin{bmatrix} \Delta S_{t-1} \\ \Delta M_{t-1} \end{bmatrix} + \begin{bmatrix} \beta_{1,3} \\ \beta_{2,3} \end{bmatrix} [\Delta Y_t^{EU}] + \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix}, \quad (53)$$

with final set of equations:

$$\Delta S_t = 0.380 + 0.018 \Delta M_{t-1} + \mathbf{0.707} \Delta \boldsymbol{S}_{t-1} - \mathbf{0.799} \Delta \boldsymbol{Y}_t^{EU} \quad (54)$$

[0.758] [0.198] [7.760] [-2.443]

R^2=0.70,

$$\Delta M_t = 0.933 + \mathbf{0.535} \Delta \boldsymbol{M}_{t-1} + \mathbf{0.319} \Delta \boldsymbol{S}_{t-1} + \mathbf{1.653} \Delta \boldsymbol{Y}_t^{EU}. \quad (55)$$

[1.488] [4.183] [2.842] [3.743]

R^2=0.82.

3.8.2 FABVARX-M

The factor augmented Bayesian vector autoregressive model with the exogenous variable as utilized for the estimation of the export variable is:

$$\begin{bmatrix} \Delta S_t \\ \Delta M_t \\ \Delta MF1_t \\ \Delta MF2_t \\ \Delta MF3_t \end{bmatrix} = \begin{bmatrix} \beta_{1,0} \\ \beta_{2,0} \\ \beta_{3,0} \\ \beta_{4,0} \\ \beta_{5,0} \end{bmatrix} + \begin{bmatrix} \beta_{1,1} & \beta_{1,2} & \beta_{1,3} & \beta_{1,4} & \beta_{1,5} \\ \beta_{2,1} & \beta_{2,2} & \beta_{2,3} & \beta_{2,4} & \beta_{2,5} \\ \beta_{3,1} & \beta_{3,2} & \beta_{3,3} & \beta_{3,4} & \beta_{3,5} \\ \beta_{4,1} & \beta_{4,2} & \beta_{4,3} & \beta_{4,4} & \beta_{4,5} \\ \beta_{5,1} & \beta_{5,2} & \beta_{5,3} & \beta_{5,4} & \beta_{5,5} \end{bmatrix} \begin{bmatrix} \Delta S_{t-1} \\ \Delta M_{t-1} \\ \Delta MF1_{t-1} \\ \Delta MF2_{t-1} \\ \Delta MF3_{t-1} \end{bmatrix} + \begin{bmatrix} \beta_{1,8} \\ \beta_{2,8} \\ \beta_{3,8} \\ \beta_{4,8} \\ \beta_{5,8} \end{bmatrix} [\Delta Y_t^{EU}] + \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \varepsilon_{3t} \\ \varepsilon_{4t} \\ \varepsilon_{5t} \end{bmatrix}, \quad (56)$$

which yields the subsequent set of equations

$$\Delta S_t = 0.529 + 0.045 \Delta M_{t-1} + \mathbf{0.666} \Delta \boldsymbol{S}_{t-1} - 0.003 \Delta MF1_{t-1} \quad (57)$$

[1.042] [0.483] [6.890] [-1.566]

$$+0.0002 MF2_{t-1} - 0.0004 \Delta MF3_{t-1} - \mathbf{0.917} \Delta \boldsymbol{Y}_t^{EU}$$

[0.263] [-0.994] [-2.703]

R^2=0.73,

$$\Delta M_t = 0.933 + \mathbf{0.514}\Delta \boldsymbol{M_{t-1}} + \mathbf{0.324}\Delta \boldsymbol{S_{t-1}} + 0.001\Delta MF1_{t-1} \quad (58)$$

[1.415] [3.928] [2.746] [0.400]

$$+0.0005MF2_{t-1} + 0.0003\Delta MF3_{t-1} + \mathbf{1.753}\Delta \boldsymbol{Y_t^{EU}}$$

[0.495] [0.558] [3.814]

R^2=0.82.

3.8.3 BVARX-X

The Bayesian estimation of the vector autoregressive model with the exogenous variable previously used for modelling the export:

$$\begin{bmatrix}\Delta S_t \\ \Delta X_t\end{bmatrix} = \begin{bmatrix}\beta_{1,0} \\ \beta_{2,0}\end{bmatrix} + \begin{bmatrix}\beta_{1,1} & \beta_{1,2} \\ \beta_{2,1} & \beta_{2,2}\end{bmatrix}\begin{bmatrix}\Delta S_{t-1} \\ \Delta X_{t-1}\end{bmatrix} + \begin{bmatrix}\beta_{1,3} \\ \beta_{2,3}\end{bmatrix}[\Delta Y_t^{EU}] + \begin{bmatrix}\varepsilon_{1t} \\ \varepsilon_{2t}\end{bmatrix}. \quad (59)$$

Provides following result:

$$\Delta S_t = 0.453 - 0.011\Delta \boldsymbol{X_{t-1}} + \mathbf{0.698}\Delta \boldsymbol{S_{t-1}} - \mathbf{0.724}\Delta \boldsymbol{Y_t^{EU}} \quad (60)$$

[0.841] [-0.108] [7.437] [-2.226]

R^2=0.70,

$$\Delta X_t = -1.149 + \mathbf{0.571}\Delta \boldsymbol{X_{t-1}} + \mathbf{0.335}\Delta \boldsymbol{S_{t-1}} + \mathbf{1.454}\Delta \boldsymbol{Y_t^{EU}} \quad (61)$$

[1.656] [4.224] [2.951] [3.369]

R^2=0.81.

3.8.4 FABVARX-X

The Bayesian estimation of the factor augmented vector autoregressive model with the exogenous variable as constructed for forecasting the export variable:

$$\begin{bmatrix}\Delta S_t \\ \Delta X_t \\ \Delta XF1_t \\ \Delta XF2_t \\ \Delta XF3_t\end{bmatrix} = \begin{bmatrix}\beta_{1,0} \\ \beta_{2,0} \\ \beta_{3,0} \\ \beta_{4,0} \\ \beta_{5,0}\end{bmatrix} + \begin{bmatrix}\beta_{1,1} & \beta_{1,2} & \beta_{1,3} & \beta_{1,4} & \beta_{1,5} \\ \beta_{2,1} & \beta_{2,2} & \beta_{2,3} & \beta_{2,4} & \beta_{2,5} \\ \beta_{3,1} & \beta_{3,2} & \beta_{3,3} & \beta_{3,4} & \beta_{3,5} \\ \beta_{4,1} & \beta_{4,2} & \beta_{4,3} & \beta_{4,4} & \beta_{4,5} \\ \beta_{5,1} & \beta_{5,2} & \beta_{5,3} & \beta_{5,4} & \beta_{5,5}\end{bmatrix}\begin{bmatrix}\Delta S_{t-1} \\ \Delta X_{t-1} \\ \Delta XF1_{t-1} \\ \Delta XF2_{t-1} \\ \Delta XF3_{t-1}\end{bmatrix} + \begin{bmatrix}\beta_{1,8} \\ \beta_{2,8} \\ \beta_{3,8} \\ \beta_{4,8} \\ \beta_{5,8}\end{bmatrix}[\Delta Y_t^{EU}] + \begin{bmatrix}\varepsilon_{1t} \\ \varepsilon_{2t} \\ \varepsilon_{3t} \\ \varepsilon_{4t} \\ \varepsilon_{5t}\end{bmatrix}, \quad (62)$$

concludes in the subsequent set of equations:

$$\Delta S_t = 0.557 + \mathbf{0.024}\Delta \boldsymbol{X_{t-1}} + 0.678\Delta S_{t-1} - 0.002\Delta XF1_{t-1} \quad (63)$$

[1.025] [0.239] [6.566] [-1.648]

$$-0.0003XF2_{t-1} + 0.0001\Delta XF3_{t-1} - \mathbf{0.829}\Delta \boldsymbol{Y_t^{EU}}$$

[-0.584] [0.993] [-2.507]

R^2=0.74,

$$\Delta X_t = 1.075 + \mathbf{0.561}\Delta \boldsymbol{X_{t-1}} + \mathbf{0.365}\Delta \boldsymbol{S_{t-1}} + 0.001\Delta XF1_{t-1} \quad (64)$$

[1.539] [4.063] [2.957] [0.663]

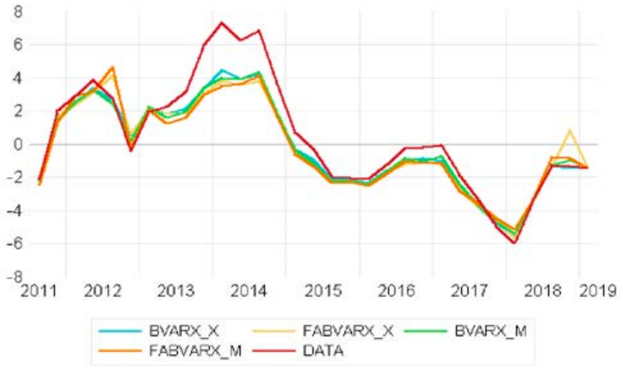
$$-0.0003XF2_{t-1} - 4.306e^{-5}\Delta XF3_{t-1} + \mathbf{1.510}\Delta \boldsymbol{Y_t^{EU}}$$

[-0.536] [-0.289] [3.433]

R^2=0.82.

Individual one-period ahead forecasts for predicting the exchange rate are summarized by the below graph.

Graph 8: Exchange Rate Forecast
YoY growth rate, in %



Source: Author’s calculations.

3.9 Labour Market Models

3.9.1 VECM Model

To forecast the labour demand variable, we construct a vector error correction model alike the one used for forecasting the GDP. This model has an advantage of outlining short-run but also long-run relationship among variables.

$$\begin{bmatrix}\Delta L_t\\ \Delta Y_t\end{bmatrix} = \begin{bmatrix}\psi_1\\ \psi_2\end{bmatrix} + \begin{bmatrix}\alpha_1\\ \alpha_2\end{bmatrix}[L_{t-1} - \beta_0 - \beta_1 Y_{t-1}] + \begin{bmatrix}\psi_{1,1} & \psi_{1,2}\\ \psi_{2,1} & \psi_{2,2}\end{bmatrix}\begin{bmatrix}\Delta L_{t-1}\\ \Delta Y_{t-1}\end{bmatrix} + \begin{bmatrix}\varepsilon_{1t}\\ \varepsilon_{2t}\end{bmatrix}. \tag{65}$$

The results of the estimation are following:

$$\Delta L_t = \mathbf{-0.094}\,(L_{t-1} - \mathbf{0.496 Y_{t-1}} - 5.103) + 0.169\Delta L_{t-1} + \mathbf{0.218 Y_{t-1}} + 0.0004 \tag{66}$$

[-2.038] [-6.172] [1.461] [3.357] [0.667]

R^2=0.58,

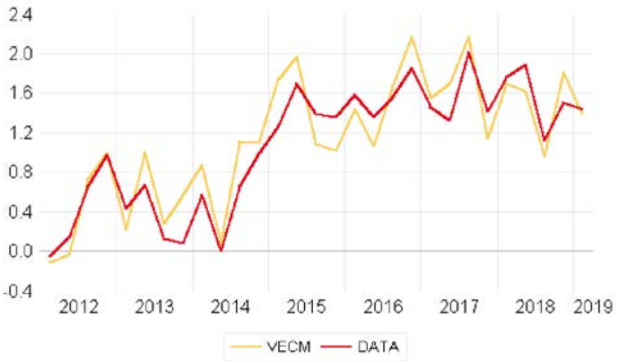
$$\Delta Y_t = -0.132\,(L_{t-1} - \mathbf{0.496 Y_{t-1}} - 5.103) - 0.369\Delta L_{t-1} + \mathbf{0.543 Y_{t-1}} + \mathbf{0.002} \tag{67}$$

[-1.150] [-6.172] [-1.277] [3.346] [1.719]

R^2=0.39.

There appears to be a clear long-run relationship between the GDP and the labour demand time series. One period ahead out-of-sample forecast for the VECM model but also for the final combined DSGE-VAR model is shown below.

Graph 9: Labour Forecast
YoY growth rate, in %



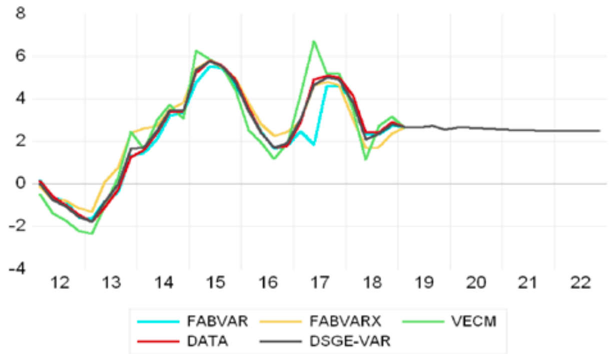
Source: Author’s calculations.

4 Final Forecast

This section offers forecasts for selected variables as obtained from the final DSGE-VAR model. The DSGE and VAR models are represented by the same weights in the final predictions. The period from 2012 to 2019 first quarter is the training period used for the one period ahead forecasts and the actual forecasts take place from 2019 second quarter to the last quarter of 2022. The outcome of the forecasting exercise is represented by the following graphs.

Graph 10: Gross Domestic Product

YoY growth rate, in %

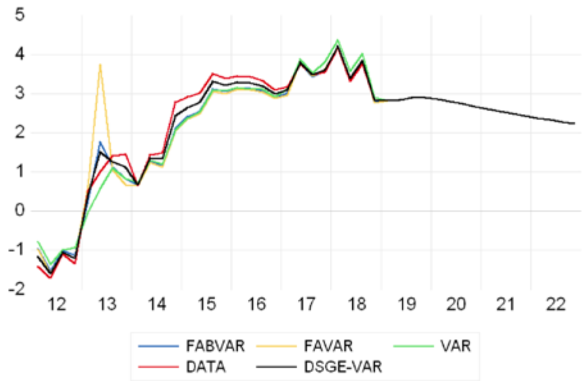


Source: Author's calculations.

4.1 GDP Decomposition

Graph 11: Consumption

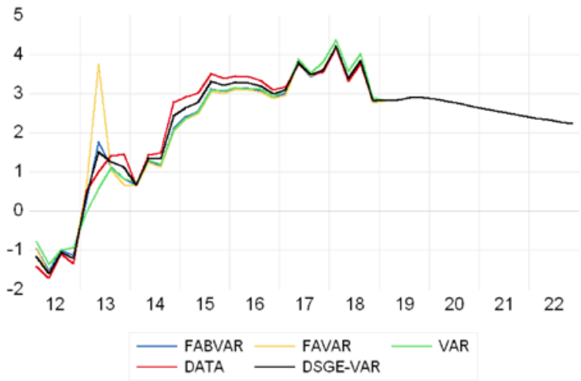
YoY growth rate, in %



Source: Author's calculations.

Graph 12: Investment

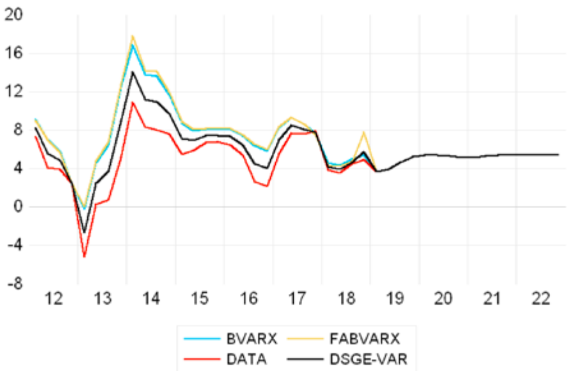
YoY growth rate, in %



Source: Author's calculations.

Graph 13: Export

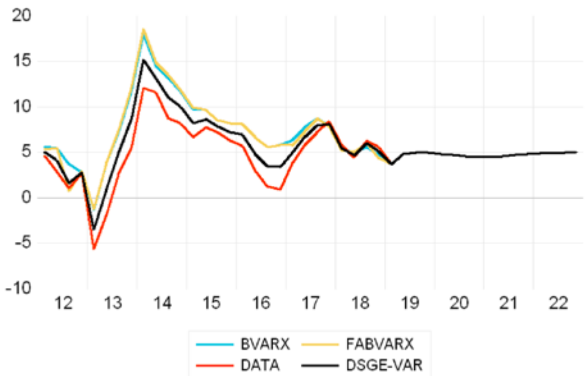
YoY growth rate, in %



Source: Author's calculations.

Graph 14: Import

YoY growth rate, in %

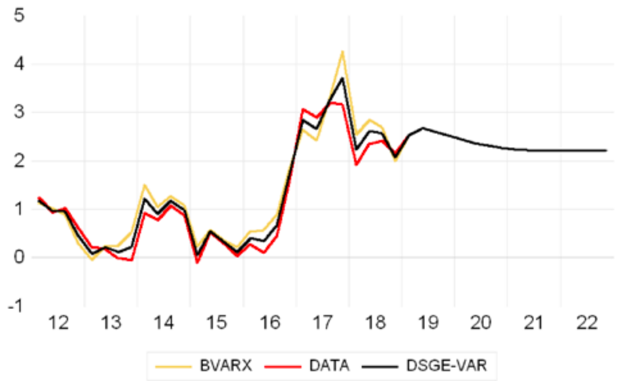


Source: Author's calculations.

4.2 Prices and Labour Market

Graph 15: Inflation

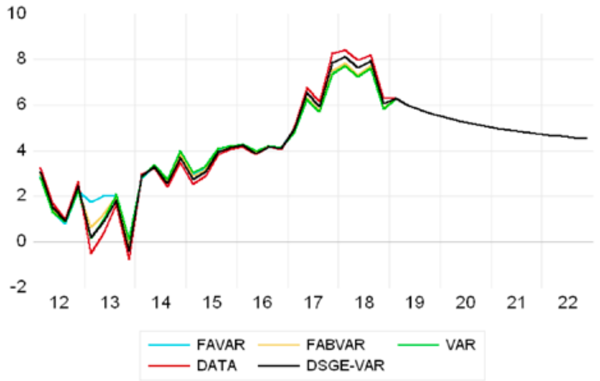
In %



Source: Author's calculations.

Graph 16: Wage

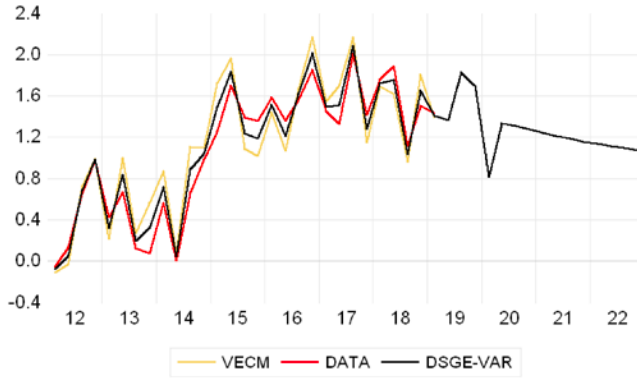
YoY growth rate, in %



Source: Author's calculations.

Graph 17: Labour

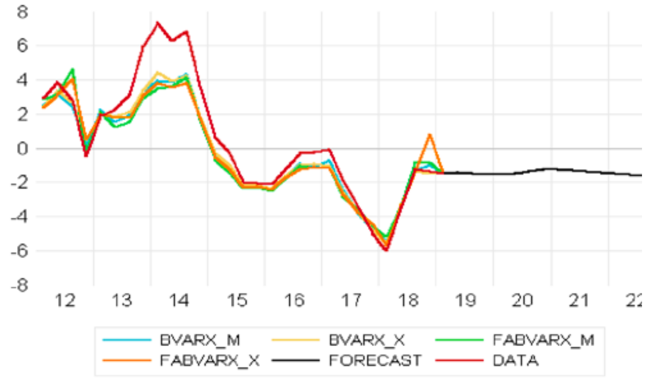
YoY growth rate, in %



Source: Author's calculations.

Graph 18: Exchange Rate

YoY growth rate, in %



Source: Author's calculations.

5 Conclusion

This study outlines VAR models for selected variables which are subsequently subject to forecast. Previous researchers argue for advantages of VAR models to obtain short-run forecasts. On the other hand, VAR models have their shortcomings, starting from a lack of deep parameters as criticized in the famous Lucas' critique to the weakness of these models to provide accurate out-of-sample forecasts while incorporating more than just few variables.

In contrast, DSGE models are attractive for their high-dimensional structure encapsulating economic linkages. The accuracy of the forecasts provided by a DSGE model is not that endangered by including multiple relationships between macroeconomic variables. Finally, DSGE models estimated by Bayesian procedure are known to serve well as medium or long-run forecasting methods, To benefit from the strength of VAR as well as DSGE models, we combine both of these model types to present a DSGE-VAR model.

The research in this paper presents the VAR component as estimated for the final DSGE-VAR model. Individual variables are estimated with selected models and then upgraded for the DSGE part before delivering final estimates for the future periods. Selection of suitable VAR models for prediction depends on economic theory, statistical significance of individual variables in the model as well as on the size of RMSE. The out-of-sample one-period ahead forecasting exercise provided for every econometric model shows them to serve as suitable tools for forecasting. These models are subsequently combined with a DSGE model utilized by the Czech Ministry of Finance to produce final forecast. The so-called DSGE-VAR model has proven to provide sensible results.

The next research could concentrate on extending VAR models for more variables.

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